A no-tension elastic–plastic model and optimized back-analysis technique for modeling nonlinear mechanical behavior of rock mass in tunneling

Cheng-Xiang Yang a,b,*, Yong Hong Wu b,*, Tung Hon b

a School of Resources & Civil Engineering, Northeastern University, Shenyang 110004, PR China
b Dept. Maths & Stats, Curtin University of Technology, GPO Box U1987, Perth 6845, Australia

1. Introduction

Over the last few decades, great effort has been made worldwide to construct robust mathematical models to study the stability of underground tunnels under various geomechanical and operational conditions. Various numerical methods such as the finite element (FE) method, the boundary element method and the limit equilibrium method have been widely used to simulate the behavior of rock masses around underground tunnels (Valiappan and Pham, 1995; Cai and Horii, 1993; Kawamoto and Aydan, 1999; Beer and Poulos, 1994; Sloan, 1989). As rock mass is one of the most complex engineering materials and the stability of geological structures generally depends on many factors, predictions of the stability of underground tunnels are extremely complicated. To produce accurate model predictions for a particular case, it is essential to construct a proper constitutive model for the rock mass.

In Western Australia, intact rock materials in most mines are fairly strong but rock masses are heavily jointed, faulted and are subjected to very high tectonic stress (ET, 1990; Keogh, 1998). For this kind of rock masses, rock failure may occur by tension or plastic yielding or combination of both forms. If the rock mass fails by tension, the tensile strength in the tension failure zone becomes zero and the stresses are transferred to other regions. Thus, to accurately describe the stability of the underground tunnels, it is necessary to model the heavily jointed rock mass as a no-tension elastic–plastic material. In this paper, we construct a set of constitutive equations capable of simulating the no-tension behavior and plastic yielding of the rock masses based on our previous work on constitutive equations capable of simulating the no-tension behavior and plastic yielding of jointed rock masses which yield according to the Drucker–Prager yield criterion and permits no-tension. A nonlinear 2-D finite element model is consequently formulated for the prediction of the behavior of the excavated rock mass. As for the model parameters, the genetic algorithm technique is employed to find the optimal rock mass properties by minimizing the discrepancy between the predicted results and field measurement. The nonlinear finite element model coupling with the genetic algorithm optimized back-analysis technique is then applied to a synthetic example of a deep tunnel in yielding rock. The results show that the forward and back-analysis system is capable of estimating the model parameters with stable and good convergence and give reasonable predictions. Numerical experiments are also carried out to check the influences of position and numbers of measurements to the reliability of the back-analysis results. Furthermore, the sensitivity analysis of the genetic algorithms optimization procedure is discussed in terms of identification of geo-material properties.
of displacements (Sakurai and Takeuchi, 1983; Gioda and Sakurai, 1987), strains (Sakurai et al., 1995), stresses (Kaiser et al., 1990), or acoustic emission information (Cai et al., 2007) during the construction stage. Because the deformation of rock masses induced by excavation can be measured easily and reliably, extensive studies have been conducted to develop displacement-based back-analysis methods (Sakurai and Takeuchi, 1983; Gioda and Sakurai, 1987; Feng and Lewis, 1987; Tang and Kung, 2009; Shang et al., 2002; Yu et al., 2007; Li et al., 2006; Zhang et al., 2006; Fakhimia et al., 2004; Ghorbani and Sharifzadeh, 2009; Hisatake and Hieda, 2008; Oreste, 2005; Vardakos et al., 2007; Yang et al., 1983; Ledesma et al., 1996; Tonon et al., 2001; Sakurai et al., 2003; Gioda, 1985; Sakurai, 1994). The numerical methods used include the finite element method, the boundary element method, the direct element method, and the finite difference method. The models range from linear elastic to nonlinear models (such as elastoplastic, visco-elastic models), from two-dimensional to three dimensional models. And the applications include different geotechnical engineering, such as tunnels, mines, dams, etc. These back-analysis methods can be roughly divided into two main categories: direct and indirect back-analysis methods. In the direct back-analysis methods, a unique relationship between field measurements and model parameters has to be derived, which is then inverted to obtain the parameter values. This approach is usually based on the assumption of linear behavior and thus it is difficult to yield a unique solution with this approach for nonlinear systems such as the cases with yielding rock masses. The indirect method, also called optimal estimation method (Feng and Lewis, 1987; Tang and Kung, 2009), uses a certain criterion to minimize the difference between the observed and calculated values at some specified observation points. This method is more flexible as it does not require establishing explicit equations relating field data to model parameters. The indirect back-analysis methods become more and more popular as advanced statistical concepts can also be included in the inverse analysis and many optimization techniques are available for the solution of the underlying optimization problem, such as the time-climbing method, the secant method, the maximum likelihood method and the Bayesian method. Recently, a kind of artificial intelligent techniques (such as neural networks and precedent type analysis) has been introduced to solve the inversion problem of displacement back-analysis and the presented results are encouraging (Shang et al., 2002; Yu et al., 2007; Li et al., 2006). In this study, the genetic algorithm (GA) Goldberg, 1989 technique is adopted to solve the inverse problem for determination of the geo-material properties, where the parameter identification is formulated as a constrained optimization problem.

Genetic algorithms are numerical search tools for finding the global optimum result of real objective functions of one or more real variables, possibly subject to various linear or nonlinear constraints. The search procedures provided by the GAs resemble certain principles of natural selection, genetics and evolution, and do not require gradient information of the objective function. Due to these advantages, the GA technique has been used extensively as a tool for solving inversion problems for parameter estimation (Goldberg, 1989; Emmanuel and Rudolf, 2004; Mareseguerra et al., 2003).

The no-tension elastic–plastic constitutive model, coupled with the nonlinear FE technique, is applied to a synthetic example of a deep tunnel in yielding rock. The importance of the rock mass properties to the tunnel deformation is discussed. The GA optimized back-analysis procedure is then employed to determine the elastic modulus, internal frictional angle and the cohesion strength from the displacements measured behind the tunnel walls. Furthermore, the sensitivity analysis of the GAs optimization procedure is discussed in terms of identification of geo-material parameters.

2. Genetic algorithms

GAs are random search algorithms based on the concepts of natural selection, genetics and evolution. The major difference between GAs and other classical optimization techniques is that GAs work with a population of possible solutions; whereas the classical optimization techniques work with a single solution. Another difference is that the GAs use probabilistic transition rules instead of deterministic rules. With these inherent advantages, GAs are more effective in finding the global optimal solutions.

In a GA search process, a group of candidate solutions, represented as genes on a chromosome in the search space, are evolved to get better solutions through natural selection and the genetic operation, crossover or recombination and mutation, borrowed from natural genetics. A chromosome can be a series of real values, binary strings, etc. Each chromosome is assigned a fitness value based on how well it performs in its environment of problem solving. Natural selection ensures that the chromosomes with better fitness values will propagate in next generations. The selection probability \( P_s \) determines the number of chromosomes to take part in genetic operations while the others die off. By using the crossover operator, the genetic information is exchanged between two selected parent chromosomes with a certain crossover probability \( P_c \), giving birth to a new chromosome (child) that has a high probability of having better fitness than its parents. Mutation allows new areas of the solution space to be explored with a probability \( P_m \). For example, if the selected parent 1 has chromosome HIJKL and the selected parent 2 has chromosome ABCDE, then one possible chromosome for the child is HICDE after the crossover operation, whereas the genes JKL on the chromosome 1 and the genes CDE on the chromosome 2 is exchanged. Then with a small probability one or more genes of the child’s chromosome will be mutated. And finally the child may have chromosome HOCDE. The process of selection/crossover/mutation is repeated to obtain a new population of candidate solutions. Successive generations are repeated until the very fit result has been found or the stop conditions are satisfied. A standard GA consists of the following steps:

1. Initialize a population of possible solutions.
2. Calculate the fitness value of each candidate solution in the population.
3. Select the solutions with higher fitness values to take part in evolution.
4. Create new population by using GA operators on the selected solutions.
5. If the prescribed stopping conditions are satisfied, then terminate the evolution process and take the evolved result with the best fitness value as the final result, otherwise, go to step 2.

The above process is shown with more details in Fig. 1. The genetic algorithms are mathematically simple yet powerful in their search for improvement after each generation (Goldberg, 1989). Due to their inherent nature, GAs have many advantages: they require no knowledge of gradient information of the objective function; high nonlinearities and discontinuities present on the objective function have little effect on overall optimization performance; they are resistant to becoming trapped in local optima; they perform very well for large-scale optimization problems and can be employed for a wide variety of optimization problems.

3. Direct problem and its numerical scheme

There are many yield criteria and constitutive models available for deformation and stress analysis of geomedia (Hill and Wu,
The simplest and generally accepted yield criterion is the Mohr–Coulomb criterion which involves only the maximum and minimum principal stresses. Various other forms of yield functions, such as the Drucker–Prager yield criterion (Chen, 1994), and have also been developed to consider the effect of intermediate principal stress. Once the stresses develop to the level meeting the yield criterion, the material changes its state from elastic to plastic and deforms according to a constitutive law. In the associated plastic flow rule theory, the increment of plastic deformation is proportional to the gradient of the yield function with respect to the stress tensor. As the Mohr–Coulomb yield function, which geometrically is a right hexagonal pyramid in the principal stress space, is not smooth, it is numerically difficult to use this yield function for the plastic deformation calculation by the associated flow rule theory. Hence in this work, we use the Drucker–Prager yield function, which is an approximation of the Mohr–Coulomb yield function in the principal stress space, for the plastic deformation analysis. As geomedia may also fail by tension or combination of both yielding and tension. In this study, the rock mass considered is assumed as a no-tension elastic–plastic continuum which yields according to the Drucker–Prager yield criterion and permits no-tension, thus:

\[ F^p(\sigma) = \eta_1 + \sqrt{j_2} - K = 0, \quad F^t = \sigma_i = 0 \]  

where \( F^p \) and \( F^t \) are the Drucker–Prager yield criterion (Chen, 1994) and the tensile failure criterion, respectively; \( I_1 \) and \( J_2 \) are the first invariant and the second invariant of the deviatoric stress tensor, respectively; \( \sigma_i \) is the principal stress component; and \( \eta \) and \( K \) are the materials constants depending on the angle of internal friction \( \phi \) and the cohesion strength \( c \) of the material.

The Drucker–Prager yield criterion in principal stress space is a smooth right circular cone equally inclined to the principal stress axes as shown in Fig. 2, and the no-tension condition cuts off all parts of the cone for which any principal stress is positive. Hence, the yield surface is the surface of a convex body OELA and all possible stress states lie on or inside the surface. The yield state depends on the location of the stress state point, and in this analysis the stress states are classified into four modes: (a) elastic state for which \( F^p < 0 \) and \( F^t < 0 \), corresponding to the regime inside the convex body OELA; (b) elastic–plastic state for which \( F^p = 0 \) and \( F^t < 0 \), corresponding to the regime on the Drucker–Prager cone surface of the convex body OELA but not including the line formed by the intersection with the tensile cut-off planes; (c) elastic–tensile failure state for which \( F^p < 0 \) and \( F^t = 0 \), corresponding to the regime on the tension cut-off planes but excluding the intersection line with the Drucker–Prager cone; (d) elastic–plastic tensile failure state for which \( F^p = 0 \) and \( F^t > 0 \), corresponding to the intersection line between the Drucker–Prager cone surface and the tension cut-off planes.

In (Wu and Schmidt, 1992), we developed a set of general constitutive equations to relate the stress increment with the strain increment, namely

\[ d\sigma = D_{ij} - D_{ij}^o d\epsilon = D d\epsilon \]  

(2)

The details of the matrices in the above equations can be found from Wu and Schmidt (1992). The above constitutive equations have been used in the study of granular flow and will be applied to rock mechanics analysis in this study. In comparison with the conventional models used in rock mechanics, the above constitutive equations are generalized equations as it includes not only the existing models (elastic model, elastic–plastic model, and elastic-no-tension model), but also a new one with both tension failure and plastic yielding occurring simultaneously.

From the principles of continuum mechanics, the stress and displacement field in rock masses are governed by the following boundary value problem,

\[ \frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0 \quad \text{in } \Omega \]
\[ \varepsilon_{ij}(\mathbf{u}) = \frac{1}{2}(\mathbf{u}_{ij} + \mathbf{u}_{ji}) \quad \text{in } \Omega \]
\[ \sigma_{ij} = D_{ijrs} \varepsilon_{rs} \quad \text{in } \Omega \]
\[ u_i = 0 \quad \text{on } \partial \Omega_2 \]
\[ \sigma_{ij} n_j = t_i \quad \text{on } \partial \Omega_2 \]

where we have used the index notation with repeated literal indices representing summation over the index range, \( \partial \Omega = \partial \Omega_1 \cup \partial \Omega_2 \) is the boundary of \( \Omega \) with \( \partial \Omega_1 \) representing the fixed boundary and \( \partial \Omega_2 \) representing the prescribed stress boundary, \( \sigma = \sigma_{ij} \) and \( \mathbf{u} \) are the stress tensor and displacement vector respectively, \( \mathbf{D} = D_{ijrs} \) denotes the constitutive matrix in Eq. (2), \( f_i \) is the body force and \( t_i \) is
the surface traction on $\partial \Omega_2$ which is related to the virgin stress field for the excavation problem.

The variational boundary value problem corresponding to the above boundary value problem is as follows. Find $u \in V$ such that:

$$a(u,v) = L(v) \quad \forall v \in V$$

(4)

where

$$a(u,v) = \int_{\Omega} (Bv)^T D(Bu) \, d\Omega, \quad L(v) = \int_{\Omega} v^T f \, d\Omega + \int_{\partial \Omega_2} v^T tds$$

$$V = \left\{ v \in [H^1(\Omega)]^2, \quad v = 0 \text{ on } \partial \Omega_2 \right\}.$$  

Using the Galerkin finite element procedure, the following discretised system of equations is obtained:

$$KU = F,$$  

(5)

where $K = [k_{ij}]$ and $F = [f_i]$ with $k_{ij} = a(\phi_i, \phi_j), \ f_i = L(\phi_i)$ in which $\{\phi_i\}_{i=1}^n$ are the basis functions of $V$.

As the constitutive matrix $D$ depends on the stress state to be determined, system (5) is a nonlinear system and thus an iteration process is needed to solve the problem. Hence, the external load is applied to the system incrementally step by step. For a typical load step, the finite element equations for the displacement increment can be obtained by using a quasi-Newton method, see for example reference (Wu et al., 2007).

4. Optimized back-analysis technique

4.1. Inverse problem for parameter identification

The purpose of back-analysis is to find the mechanical parameters of rock masses that result in good agreement between calculation results and field measurements. Displacements, local stresses and strains at some selected points in the rock mass around the openings constitute typical field measurements. The inverse problem for parameter estimation can be defined as the following constrained optimization problem:

Minimize $F(p)$ Subject to

Constraints $\mathbf{p} \in \mathbb{R}, p_{j,\text{min}} \leq p_j \leq p_{j,\text{max}} \quad j = 1, 2, \ldots$

where $\mathbf{p} = (p_1, p_2, \ldots)^T$ is the vector of unknown parameters; $F(\mathbf{p})$, called the cost function or objective function, is usually defined based on the discrepancy between the quantities $u_i$ measured in the field and the corresponding data $u_i$ obtained by the forward analysis of a representative calculation model. For example, the mean square error is widely used as the objective function, namely

$$F(p) = \frac{1}{n} \sum_{i=1}^{n} [u_i(p) - u_i]^2$$

(6)

where $n$ is the number of measurements. This function is minimized with respect to the unknown parameters $\mathbf{p}$ that influences the behavior of the mechanical problem, to find the optimal values of the mechanical properties for the rock masses.

The objective function defined by Eq. (6) is a highly nonlinear function of the parameters $\mathbf{p}$. Since an analytical expression of $F$ cannot be defined, it is very difficult to obtain an analytical evaluation of the function gradient. These features of the objective function make the back-analysis procedure very costly, especially in elasto-plasticity analysis (Gioda, 1985; Sakurai, 1994). In this case, GAs provide powerful alternatives.

4.2. Solution of the inverse problem using genetic algorithms

In order to carry out a GA search to solve the inverse problem, it is necessary to define an evaluation function that returns a measurement of the fitness of any chromosome in the context of the problem. Unlike the standard GA that uses a profit value to represent the fitness, in this study, the fitness is measured by a cost value based on the discrepancy between the calculation results and field measurement, because such a cost value represents the error and thus the analysis is more suitable for the current back-analysis problem. Hence, the minimum value of the fitness function corresponds to a better chromosome. The fitness function can be defined as the objective function or its varieties. Here the root of the mean square error is used, namely

$$\text{Fitness} = \left( \frac{1}{n} \sum_{i=1}^{n} [u_i - u_i^*]^2 \right)^{1/2}$$

(7)

With the measurable quantities $u_i$ at selected measuring points computed by means of the FE method in the preceding section, the GA loop with the fitness defined above can now be carried out to find the optimal set of boundary stress parameters from the field measurement in an iterative manner. The solution procedure starts from a population of candidate solutions $\mathbf{p}_j (k = 1, 2, \ldots, N)$, where $N$ is the population size. Each possible solution is used as real mechanical parameters of rock masses for the FEM model to predict the boundary value of $u_i$ at the observation points. The $u_i$ obtained from the FE computation is then compared with those $u_i$ obtained from the field measurement to evaluate the fitness of the population of the candidate solutions by Eq. (7). The estimated fitness values are then used by the GA operations to obtain a new population of the parameter sets and thus evolve into a new generation. A typical crossover operator in the context of parameter estimation is shown in Fig. 3. The population of candidate parameter sets is updated until the prescribed stopping criterion is met. The GA thus acts as an error minimization filter in the loop and the FE acts as an estimation tool. By introducing the notations FE and GA to represent the two functional parts of the loop respectively, the iterative procedure of the optimized inversion analysis method can be briefly formulated as follows.

Given initial parameter sets $\mathbf{p}_j (j = 1, 2, \ldots)$ and measurements $\mathbf{u}^*$.

Starting from the first generation $t = 0$, perform the following steps repeatedly until the stopping criteria are met:

$$\mathbf{u}_t(p_j) = \text{FE}(p_j)$$

$$\mathbf{F}_t = F_t(p_j) = \left\{ \frac{1}{n} \sum_{i=1}^{n} [\text{FE}(p_j) - u_i]^2 \right\}$$

$$\mathbf{p}_{t+1} = \text{GA}(\mathbf{p}_t, \mathbf{F}_t)$$

**Fig. 3.** Crossover operation of two candidate parameter sets during the GA optimized parameter estimation procedure ($f(\mathbf{p})$ is some crossover rule).
The flow chart of the solution procedure of using the GA coupled with the FE solver is shown in Fig. 4.

5. Numerical example

A comprehensive evaluation of the proposed constitutive model and the GA optimized back-analysis system was carried out for the elasto-plastic case in a hypothetical site. As a synthetic example, we consider the excavation of a simple-shaped deep tunnel as shown in Fig. 4. The rock mass is assumed as the above-mentioned no-tension elastic–plastic continuum which yields according to the Drucker–Prager yield criterion and permits no-tension. Its relevant mechanical properties are listed in Table 1.

For simplicity, the virgin state of the in situ stress state is assumed to be known (see Fig. 5). The major and minor principal stresses act on the plane orthogonal to the tunnel axis and are equal to 25 MPa and 13 MPa, respectively. The major principal stress is inclined 18° below the horizontal, while the minor principal stress in inclined 72° above the horizontal. A Fortran 90 code capable of coupling the nonlinear FE analysis and the GA iteration procedure was used in the following analysis.

5.1. Sensitivity analysis of rock mass properties

Sensitivity analysis is done to check the change in the tunnel displacement due to perturbations in the geo-material parameters. The three main mechanical properties of yielding rock mass around the tunnel, elastic modulus $E$, cohesion strength $C$ and internal friction angle $\phi$ of the rock mass are investigated. The testing values of the parameters are selected around the real parameter values given in Table 1. In practice, a reasonable range for each of the parameter values can be selected according to the results of field tests and engineering judgment. During the sensitivity analysis for a chosen parameter, the other parameters are fixed with the values given in Table 1, and the nonlinear FE analysis formulated in Section 3 is carried out to calculate the displacement at the observation points set at the crown and the midpoint of the right sidewall of the tunnel. The variations in both the vertical and horizontal components of the displacement with the parameter are then compared.

Figs. 6–8 show the results of the sensitivity analysis. It can be seen that the tunnel displacement depends on the elastic modulus when the material is in elastic state. If plastic deformation occurs, elastic modulus and internal friction angle are major influencing factors to the tunnel displacement. It is also noted that the cohesion does not affect the tunnel displacement significantly.

5.2. Estimation of rock mass properties

The no-tension elasto-plastic FE model is adopted for the GA optimized back-analysis to obtain the optimal parameters of in situ principal stresses which result in good agreement of predicted displacement with the hypothetical field measurement. In a tunnel monitoring program, the measurement of tunnel wall displacements is generally involved. We assume that the measured quantities are the tunnel wall absolute displacements. Both the vertical and horizontal components of the displacements are used. The measuring points and observed values of the tunnel displacements are shown in Fig. 5 and Table 2. The elastic modulus, internal frictional angle and the cohesion strength of the rock mass are parameters to be determined.

In the present analysis, a real-coded GA in which all genes in a chromosome are real numbers has been used, since it is more suitable and convenient to directly represent genes as real values for most real-optimization problems during genetic operations without coding and decoding the design variables. The real-coded GA with a reset stochastic selection type of selection procedure, simulated binary crossovers and polynomial mutations has been used. For details of these genetic operators, we refer the reader to (Deb, 2001). The parameters for the real-coded GA are selected as follows: the population size $N = 30$, the allowed maximum number of generations $M = 200$, the crossover probability $= 0.7$, and the mutation probability $= 0.2$. The value of $p_c$ or $p_m$ is used as upper limit to decide how a new candidate solution will be generated.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Properties of the rock mass.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight (KN/m$^3$)</td>
<td>28</td>
</tr>
<tr>
<td>Elastic modulus (MPa)</td>
<td>2700</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.23</td>
</tr>
<tr>
<td>Failure criterion</td>
<td>Drucker–Prager</td>
</tr>
<tr>
<td>Tensile strength (MPa)</td>
<td>0.1</td>
</tr>
<tr>
<td>Cohesion strength (MPa)</td>
<td>0.35</td>
</tr>
<tr>
<td>Internal friction angle (°)</td>
<td>30</td>
</tr>
</tbody>
</table>

Fig. 5. Mesh, measurement points and in situ stress state of the numerical example.
by comparing with a uniform random number on the interval (0, 1) preassigned to the candidate solution. That is, in the current application, about 70% of the new population is randomly selected to be generated through crossover operation, while 20% of the new population is randomly selected to be mutated. We terminate the process when the best solution in the population remains unchanged over several generations. According to Section 4, the rock mass properties estimation process can be outlined as shown in Fig. 9.

In order to check the effect of the number of measuring points on the back-analysis system, numerical experiments with varying measurements are carried out during the analysis. The positions of the points are selected according to the following three

Fig. 6. Effect of elastic modulus on tunnel displacement.

Fig. 7. Effect of cohesion strength on tunnel displacement.

Fig. 8. Effect of internal friction angle on tunnel displacement.
Average performances of back-analysis with different configurations of measurements.

Table 3

<table>
<thead>
<tr>
<th>Measuring point</th>
<th>Measured (assumed) displacements (mm)</th>
<th>Real values</th>
<th>( E ) (MPa)</th>
<th>( C ) (MPa)</th>
<th>( \varphi ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal component</td>
<td>Vertical component</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>10.70</td>
<td>–3.38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>–5.39</td>
<td>–0.72</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>–2.12</td>
<td>0.82</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.02</td>
<td>0.49</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.85</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10.33</td>
<td>46.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>9.65</td>
<td>44.38</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8.23</td>
<td>39.98</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>7.39</td>
<td>35.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>6.18</td>
<td>28.72</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

We can see that the best fitness (minimum objective function) is achieved using the above-mentioned scheme (iii), i.e., points 1, 2, 6, 7 (see Fig. 5). We can determine the best fitness (minimum objective function) is...
almost constant after certain number of generations. That is, stable convergence is obtained by using the GA optimized back-analysis technique. It can also be seen that there is sufficient diversity in the results as observed from worst and average fitness. This is one of the most important aspects in GA that can guarantee the optimality of the results. Accordingly, the variation of the estimate parameters with generation numbers is plotted in Fig. 11. Stable convergence can be seen again and the obtained parameters agree
with the actual values very well. From the tuning trajectories of the three estimate parameters and the number of generations used in different runs it can be concluded that the algorithm can find the ways to the global optimal results from different start points and therefore gives reliable solutions. In Fig. 11, it can easily be observed that the convergence is not reached simultaneously or randomly by all parameters. By comparing several runs of the back-analysis processes with different random initial populations, we found that convergence of the elastic modulus $E$ and the internal friction angle $\phi$ were achieved much earlier than that of the cohesion strength $C$. This can be attributed to the importance of their contributions to the objective function, namely the excavated displacements since they are the only variables of the objective function, as is consistent with the results of the above sensitive analysis.

The computed displacements at the measuring points and other two testing points A and B (seen in Fig. 5) during the iterative estimation process are shown in Figs. 12 and 13. Similar convergence can be seen in displacement values. The comparison of final predicted results with measurements is given in Table 4. There is a good agreement between the predicted and measured displacements. This indicates that the proposed method find the global optimal solutions which can not only reproduce the results at the measuring points, but also give reasonable prediction of deformation in other areas around the tunnel.

### 6. Sensitivity analysis of the optimized back-analysis technique

Like most other population based optimization methods, the genetic algorithm proceeds by adjusting first the parameter which is the most sensitive to the objective function, worrying only at a second stage about the tuning of the other less important variables which can give only minor improvements of the fitness function value (Marseguerra et al., 2003). Marseguerra et al. (2003) have shown that for GAs, the sensitivity of the parameters can be correlated with the square of the coefficient of variation. The coefficient of variation is defined as the ratio of the standard deviation to the mean of the population. The parameter which is the most sensitive should quickly converge to a low value of the square of coefficient of variation with the increase of generation. To perform the sensitivity analysis, an archive of the currently best 200 individuals

### Table 4
Prediction results using the estimated parameters.

<table>
<thead>
<tr>
<th>Point no.</th>
<th>Horizontal component (mm)</th>
<th>Vertical component (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Measured</td>
<td>Predicted</td>
</tr>
<tr>
<td>1</td>
<td>10.70</td>
<td>10.7084</td>
</tr>
<tr>
<td>2</td>
<td>-5.39</td>
<td>-5.40385</td>
</tr>
<tr>
<td>6</td>
<td>10.33</td>
<td>10.32454</td>
</tr>
<tr>
<td>7</td>
<td>9.65</td>
<td>9.652131</td>
</tr>
<tr>
<td>A</td>
<td>18.12</td>
<td>18.12644</td>
</tr>
<tr>
<td>B</td>
<td>-6.27</td>
<td>-6.21536</td>
</tr>
</tbody>
</table>

**Fig. 13.** Variation of the prediction results with generation numbers at testing points.

**Fig. 14.** Behaviors of the square of the coefficient of variation as a function of the generations in two typical runs.
found by the algorithm was filled and updated. At the end of each generation we then evaluated the sample means and variances of the three parameters from the respective 200 values contained in the current archive. The behavior of the square of coefficient of variation with generation is shown in Fig. 14. It can be seen that the GA first optimizes the elastic modulus $E$, then the internal friction angle $\phi$ and finally the cohesion strength $C$, which is consistent with the results of Fig. 11 as well as the results of sensitivity analysis of rock properties. Furthermore, it can be concluded that the tunnel displacement is more sensitive to the internal friction angle than the elastic modulus of the rock mass while excavating in yielding rock mass.

7. Conclusion

This paper presents a forward and back-analysis system coupling nonlinear finite element analysis and optimized back-analysis technique for stability analysis of underground tunnels. A no-tension elastic-plastic constitutive model is applied to simulate the no-tension behavior and plastic yielding of jointed rock masses and an iterative procedure based on the genetic evolutionary search technique is proposed to find the optimal geo-material parameters.

A simple synthetic example was analyzed with reference to a deep tunnel excavated in a yielding rock mass. A sensitivity analysis of the rock mass properties shows that the tunnel displacement significantly depends on the elastic modulus and internal friction angle and is not significantly affected by the cohesion strength. Since no knowledge on gradient information of the objective function is required, the optimized back-analysis procedure has high reliability in the presence of highly nonlinear yielding behavior of rock mass. Besides estimating efficiently the values of the parameters, the optimization approach also provides a qualitative indication of the importance of different parameters with respect to the objective function. Numerical experiments show that the numbers and positions of the measuring points play a crucial role in the back-analysis system and should be carefully selected and the sufficiency as well as the diversity of displacement information should be considered when designing the measuring system.

Acknowledgements

Financial support from the Australian Research Council under Linkage Project Grants, the National Nature Science Foundation of China under Grant No. 50504004, the Program for New Century Excellent Talents in University, and the Special Funds for Major State Basic Research Project under Grant No. 2010CB732000 is gratefully acknowledged.

Reference


